Beta Decay of Carbon 10 and the Cluster Model*†

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(Received 8 July 1963)

A new toroidal spectrometer has been used to determine an end point of 1865±15 keV for the main positron branch in the decay of C^{10}. This end point, together with the previous work on the mass 10 nuclei, leads to a Coulomb energy difference of 4.61 MeV between the mirror nuclei C^{10} and Be^{10}. According to the cluster model, developed by Wildermuth and Kanellopoulos, C^{10} is composed of a di-proton cluster and two alpha-particle clusters. A lower bound on the size of the di-proton cluster may be obtained from the Li^6 charge radius, while the size of the alpha clusters can be calculated from the charge radius of C^{12}. However, these estimates of cluster size imply an upper limit of 3.52 MeV on the $C^{10}-Be^{10}$ Coulomb energy difference. Hence, it is concluded that this particular cluster model does not describe the spatial properties of these nuclear clusters in a manner consistent with experimental evidence.

I. INTRODUCTION

THE beta decay of C^{10} was first investigated by Sherr, Muether, and White.[1] They showed that C^{10} decays with a half-life of 19.1±0.8 sec. By means of absorption in aluminum, they determined the maximum energy of the positrons to be 2.10±0.10 MeV. Later, Sherr and Gerhart[2] studied the gamma radiation associated with the beta decay of C^{10}. Their results indicated that the C^{10} beta spectrum has at least two components: One group of relative intensity 98.4% and end point about 2.1 MeV arises from a transition to the first excited state of B^{10}; another group of relative intensity 1.65±0.20% comes from a transition to the second excited state of B^{10}.

The purpose of this paper is twofold. First, it is intended to describe measurements of the beta spectrum and half-life of C^{10}, and second, to interpret the present information on the lowest isotopic triplet in the mass 10 nuclei.

By means of a new toroidal spectrometer,[3] the end point of the principal beta group was found to be 1 865±15 keV. Although this result contradicts the previous measurement of 2.10±0.10 MeV, it agrees with a recent $B^{10}(p,n)C^{10}$ threshold determination by Takayanagi *et al.*[4] In addition, a half-life measurement was made in order to establish that only one radioactive species had been involved in the end-point work. The experimental decay curves were well fitted by a single component with a half-life of 19.27±0.08 sec. This value, however, differs from the 19.48±0.05 sec recently reported by Earwaker *et al.*[5]

Figure 1 shows the beta decay scheme of the mass 10 nuclei. As noted, three of the states involved form an isotopic spin triplet. This assignment, of course, assumes the charge independence of nuclear forces. The following analysis, however, depends only on the charge symmetry of nuclear forces. This assumption allows one to calculate a Coulomb energy difference of 4 615±19 keV from the difference in the binding energies of C^{10} and Be^{10}. For any given form of the nuclear charge distribution, the $C^{10}-Be^{10}$ Coulomb energy difference determines the nuclear charge size. On the other hand, if the charge independence of nuclear forces in invoked, the form of the nuclear charge distribution enables one to predict the location of the middle member of the isotopic triplet. A comparison of this prediction with the observed location provides an estimate of the validity of the charge-independence hypothesis.

In addition, the $C^{10}-Be^{10}$ Coulomb energy difference, together with the Li^6 charge distribution measured by Burleson and Hofstadter,[6] provides a test of the cluster model of the nucleus proposed by Wildermuth and Kanellopoulos.[7] In this model nuclei are pictured as groups or clusters of nucleons moving in an average potential well. For example, Li^6 is composed of a deuteron cluster and an alpha-particle cluster, while C^{10}

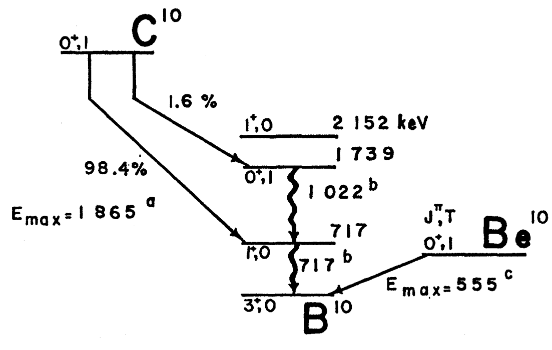

FIG. 1. Decay scheme of the mass 10 nuclei. (a) Present work. (b) Data of V. K. Rasmussen, W. F. Hornyak, and T. Lauritsen, Phys. Rev. 76, 581 (1949). (c) Data of C. W. Li, W. Whaling, W. A. Fowler, and C. C. Lauritsen, Phys. Rev. 83, 512 (1951).

* Work supported in part by the U. S. Atomic Energy Commission.
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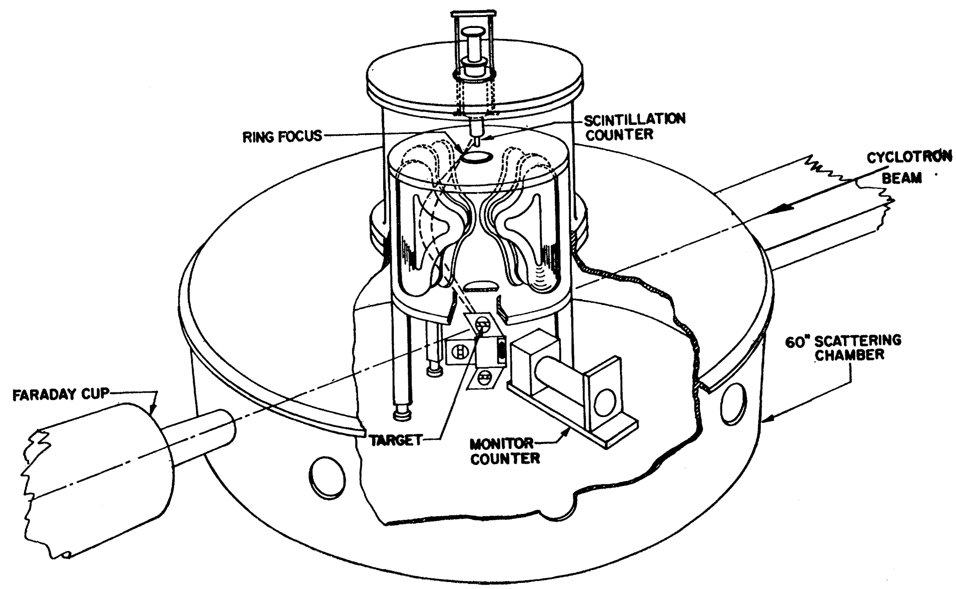

FIG. 2. Schematic drawing of the experimental setup for the measurement of the C^{10} end point. The 38 gap toroidal spectrometer is mounted above the target in a large scattering chamber. For simplicity, only four gaps are shown.

consists of two alpha-particle clusters (or a Be^8 cluster) and a di-proton cluster. According to Wildermuth and Kanellopoulos, the observed long-tailed charge distribution of Li^6 reflects the large size of the deuteron cluster. Likewise, a large di-proton cluster in C^{10} should be evident from the $C^{10}-Be^{10}$ Coulomb energy difference. However, if the di-proton cluster in C^{10} is taken equal in size to the deuteron cluster in Li^6, a Coulomb energy difference of 3.52 MeV is found, whereas the experimental value is 4.61 MeV. Hence, it is concluded that the cluster model does not give a consistent account of the spatial properties of these nuclear clusters.

II. MEASUREMENTS

A. Procedure and Equipment

Figure 2 shows the experimental arrangement for the measurement of the beta spectrum of C^{10}. The 38 gap toroidal or iron-free orange spectrometer was mounted in a large scattering chamber in the experimental area. C^{10} was produced by the $B^{10}(p,n)C^{10}$ reaction in a B^{10} target located in the source position of the spectrometer. Because of the 19-sec half-life of C^{10}, the beam was programmed off and on during alternate 15-sec intervals. During the beam-off periods, the positrons from the decay of C^{10} were analyzed by the toroidal spectrometer, while the total beta activity of the target was determined by a monitor scintillation counter. At each current setting of the spectrometer, the background counting rates were also measured. This background arose both from long period activities present in the scattering chamber and from gamma rays associated with the beta decay of C^{10}. To estimate the contribution of the latter, the program of bombarding and counting was repeated but with a thick absorber over the detector of the spectrometer.

In the course of the present work, two types of B^{10} targets have been used. For one, a thin 6 mm×8 mm wafer was made from 96% enriched boron powder and mounted in a target frame as shown in Fig. 3. Since this type of target was very fragile, slurry targets were also employed. For these, a slurry of 97% enriched boron powder and distilled water was deposited on a thin Mylar foil and allowed to dry.

For the production of C^{10}, the boron targets were bombarded with 10.5-MeV protons from the hydrogen molecular-ion beam of the University of Washington 60-in. cyclotron. In order to insure that the beam was properly centered and uniformly spread on the target, a fluorescent screen was placed at the target position

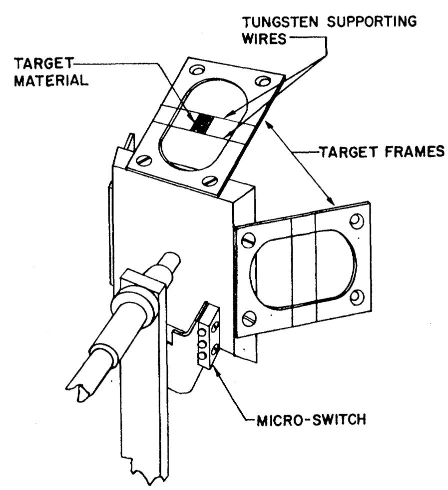

FIG. 3. Schematic drawing of the target "windmill." A B^{10} pressed powder or wafer target is shown at the top. The blank target frame on the right is used to ascertain the background from the tungsten support wires.

and observed by means of a closed circuit television system. Occasionally a blank target was bombarded. No radiation was observed from the tungsten support wires.

Beside the targets shown in Fig. 3, the target "windmill" held a Cs^{137} calibration source. The 624-keV internal conversion electrons were used to calibrate both the toroidal spectrometer and the monitor scintillation counter.

The monitor counter consisted of a 19-mm-diam×10-mm-thick Pilot B plastic scintillator mounted directly on a Dumont-6291 photomultiplier. A 240-mg/cm² aluminum foil was placed in front of the scintillator to absorb out the large number of scattered protons during the beam-on periods. Because C^{11} was made from the B^{11} impurity in the target material by the $B^{11}(p,n)C^{11}$ reaction, only pulses corresponding to beta particle energies above 1 MeV were accepted in the monitor counting system. To reduce the background radiation level, the scintillator was shielded on its sides by about 3 cm of lead.

In the early stages of this work, a Geiger counter served as the detector of the spectrometer. Later, the scintillation counter shown in Fig. 2 was installed. In this counter, a 19-mm-diam×63-mm-high Pilot B plastic scintillator was coupled through a lucite light pipe to a Dumont-6291 photomultiplier.

B. End Point

After corrections were applied to take account of the background radiation, each counting rate measured in the spectrometer was normalized to the counting rate in the monitor counter. Fermi-Kurie plots, such as that shown in Fig. 4, were constructed from the normalized counting rates. The Fermi function was evaluated from the Bureau of Standards tables.[8] The result, obtained by linear extrapolation in twelve Fermi-Kurie plots, was an end point of 1 865±15 keV for the principal branch in the beta decay of C^{10}. The uncertainty in the result arose mainly from the target thickness and the gamma-ray background in the spectrometer.

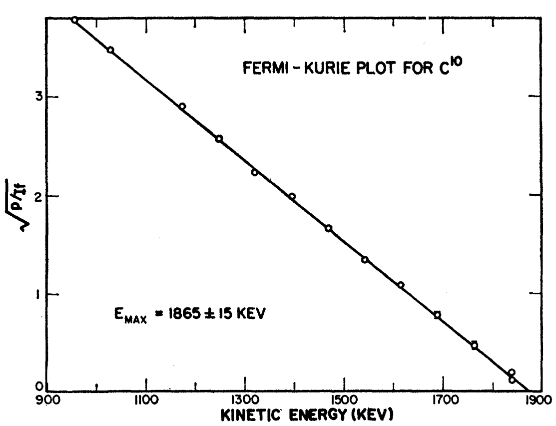

FIG. 4. Fermi-Kurie plot for the C^{10} beta spectrum.

[8] U. Fano, Tables for the Analysis of Beta Spectra, National Bureau of Standards Applied Mathematics Ser. No. 13, (1952).

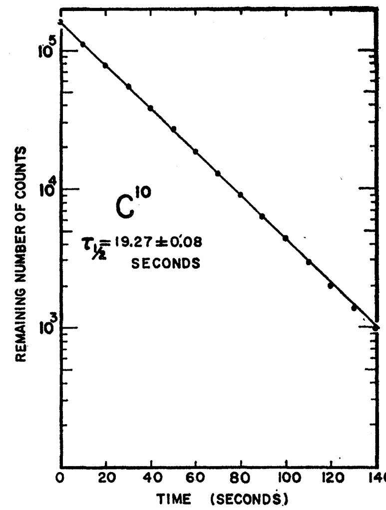

FIG. 5. Radioactive decay of C^{10}. The experimental data were analyzed by a method due to Kuscher *et al.* (see Ref. 10).

After the completion of the end-point measurement described above, Takayanagi *et al.*[4] reported a threshold of 4 835±25 keV for the $B^{10}(p,n)C^{10}$ reaction in good agreement with the 4 827±16 keV computed from the beta-decay data.

C. Half-life

The half-life of C^{10} was determined by observing the decay in counting rate in the monitor scintillation counter. A half-life run was initiated automatically when the cyclotron beam was turned off. By means of an electronic time-to-pulse-height converter,[9] the decay of the C^{10} activity was followed for about 10 half-lives. Background measurements were made before and after each run. In no run was the background level more than 2% of the initial counting rate.

The half-life of C^{10} was found from these data by a graphical method developed by Kuscher *et al.*[10] For this, the remaining number of counts was computed and plotted as a function of time. Such a plot is shown in Fig. 5. The result was a half-life of 19.27±0.08 sec for the decay of C^{10} in disagreement with the 19.48±0.05 sec obtained by Earwaker *et al.*[5]

III. THEORETICAL IMPLICATIONS

A. Charge Independence

Figure 1 shows the beta-decay scheme of the mass 10 nuclei. In order to relate this information to some basic concepts, the charge symmetry of nuclear forces is invoked. Accordingly, after correction for the neutron-proton mass difference the difference in the binding

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[10] I. Kuscher, M. V. Mihailovic, and E. C. Park, Phil. Mag. 2, 998 (1957).

TABLE I. Experimental and theoretical Coulomb energy differences (keV).[a]

Nuclei	Experiment	Oscillator L-S coupling	Oscillator j-j coupling	Classical method
$C^{10}-B^{10**}$	2648±17	2629	2603	2854
$B^{10**}-Be^{10}$	1967± 5	1986	2012	1789
$O^{14}-N^{14*}$	3619± 4	3604	3540	3627
$N^{14*}-C^{14}$	2939± 2	2954	3018	2931

[a] The predictions in the third and fourth columns were based on the oscillator model calculations of Carlson and Talmi (Ref. 11); those in the last column were obtained by a classical method emphasized by Wilkinson (Ref. 12). All the data used (except the C^{10} end point) were taken from F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. **11**, 1 (1959).

energies of C^{10} and Be^{10} is interpreted as a Coulomb energy difference of 4615±19 keV.

The first concept to be examined is the more restrictive charge independence of nuclear forces. Under this hypothesis there exists a state in B^{10} which is described by the same nuclear potential and the same space-spin wave function as the ground states of C^{10} and Be^{10}. Hence, its position relative to these levels is determined solely by the Coulomb energies of the fifth and sixth protons. If a harmonic oscillator well is adopted for the nuclear potential, these Coulomb energies can be obtained in terms of the oscillator parameter $\nu \equiv m\omega/\hbar$ from the work of Carlson and Talmi.[11] Since the empirical $C^{10}-Be^{10}$ Coulomb energy difference can be expressed in the same form, the position of the analogous state in B^{10} can be readily predicted.

In Table I, the experimental Coulomb energy differences for $A=10$ are compared with those obtained by means of the shell-model calculations of Carlson and Talmi under the hypothesis of charge independence. A similar analysis has been carried out for the lowest isotopic spin triplet at $A=14$. In addition, Table I contains the results from a classical approach which has been emphasized by Wilkinson.[12] His method starts also from the assumption of charge symmetry but corrects the empirical Coulomb energy differences of the neighboring odd mass mirror nuclei for the change in nuclear radius in going to the isotopic triplet nuclei. It is not surprising that this classical approach, which neglects the exchange energy, does not explain the experimental facts in a consistent fashion. However, the results based on the shell model with the more appropriate L-S coupling suggest that the nuclear potential is charge independent to about 20 keV or 0.4%. Although this estimate is in line with the strength of the electromagnetic interaction relative to the nuclear interaction ($\approx$0.7%), still the large uncertainty in the C^{10} end point, the use of an *infinite* harmonic oscillator well, and the neglect of radiative corrections render the quantitative value of this estimate quite doubtful.

[11] B. C. Carlson and I. Talmi, Phys. Rev. **96**, 438 (1954).
[12] D. H. Wilkinson, Phil. Mag. **1**, 1031 (1956).

B. Nuclear Charge Size

The $C^{10}-Be^{10}$ Coulomb energy difference also enables one to determine the charge size of nuclei. For example, one can evaluate the oscillator parameter ν in the expression for the $C^{10}-Be^{10}$ Coulomb energy difference given by Carlson and Talmi.[11] In turn, assuming that the oscillator parameter is independent of the mass number, one can calculate the charge radii of some neighboring stable nuclei. According to the virial theorem, a particle in an oscillator well with energy $\frac{1}{2}(2n+3)\hbar\omega$ has a mean-square radius of $(2n+)3/2\nu$. Therefore, the mean-square charge radius is found by averaging $(2n+3)/2\nu$ over all the protons. The results of these calculations are presented in Table II. This table also includes the values which Meyer-Berkhout *et al.*[13] computed from the electron scattering data using the same functional form for the nuclear-charge distribution. In this mass region these two charge-sensitive

TABLE II. Nuclear charge radii (in F) calculated from the Stanford electron scattering measurements [see Ref. 13] and from the $C^{10}-Be^{10}$ Coulomb energy difference on the basis of a harmonic oscillator model with L-S or j-j coupling.

Nucleus	Electron scattering	Oscillator L-S coupling	Oscillator j-j coupling
C^{12}	2.41	2.41	2.30
Be^{9}	2.26	2.32	2.29

methods of determining nuclear radii seem to be in substantial agreement. On the other hand, Sengupta[14] used the $Be^{9}-B^{9}$ Coulomb energy difference in a similar calculation and found a charge radius of 2.48 F for Be^{9}. However, this discrepancy can probably be attributed to a large Thomas–Ehrman shift in B^{9} which is unstable to proton emission.[12,15]

C. Nuclear Cluster Size

Wildermuth and Kanellopoulos[7] have developed their cluster model of the nucleus as an improvement of the simple shell model. Nuclei are pictured in this model as groups or clusters of nucleons moving in an average potential well. From an analysis of many light nuclei Wildermuth and Kanellopoulos[16] have concluded that the clusters present in nuclei closely resemble the free clusters. Conversely, this similarity has been used to obtain a simple physical insight into many nuclear properties[7] and reactions.[17] For example, Burleson and Hofstadter[6] have observed a long-tailed charge distribution of radius 2.82 F for Li^{6}. Further, their measure-

[13] U. Meyer-Berkhout, K. W. Ford, and A. E. S. Green, Ann. Phys. (N. Y.) **8**, 146 (1959).
[14] S. Sengupta, Nucl. Phys. **21**, 542 (1961).
[15] P. Goldhammer, Rev. Mod. Phys. **35**, 40 (1963).
[16] K. Wildermuth and Th. Kanellopoulos, Nucl. Phys. **7**, 150 (1958).
[17] G. C. Morrison, Phys. Rev. Letters **5**, 565 (1960).

ments could not be fitted on the shell model with a single oscillator well, even though this scheme had been satisfactory for other 1*p*-shell nuclei. On the other hand, according to the cluster model Li^6 in the ground state consists of a deuteron cluster and an alpha-particle cluster.[18] The smearing observed in the charge distribution is simply caused by the large size of the deuteron cluster.

A similar approach can be used in an attempt to understand the $C^{10}-Be^{10}$ Coulomb energy difference. However, in this case the cluster model fails to describe the nuclear clusters in a manner consistent with the experimental evidence. According to the cluster model, C^{10} (or Be^{10}) should be regarded as a di-proton (or a di-neutron) and two alpha-particle clusters. From the observed properties of the free di-nucleon, it is expected that a di-proton or di-neutron cluster is no smaller than a deuteron cluster. Accordingly, a lower limit on the size of the di-proton cluster may be obtained from the deuteron cluster in Li^6. However, the electrostatic energy of such a di-proton cluster in C^{10} is only 3.52 MeV as compared with the $C^{10}-Be^{10}$ Coulomb energy difference of 4.61 MeV.

In the computation of the Coulomb energy of the di-proton cluster in C^{10} the spirit of the cluster model was followed as closely as possible. The cluster-model wave function was explicitly written down in terms of shell-model wave functions according to the prescription of Kanellopoulos and Wildermuth.[19] The oscillator well for the alpha-particle clusters was chosen so as to give C^{12} a rms radius of 2.41 F. In effect, the enlarged di-proton cluster was represented as two "lp" protons moving in a broader well than the other nucleons. Its Coulomb energy was computed in this representation by an approximate method suggested by Thieberger.[20] Thus, the direct Coulomb integrals for two protons, one moving in an oscillator well characterized by ν_1 and the other in a well characterized by ν_2, were calculated as if both protons were in an oscillator well described by $\nu=2\nu_1\nu_2/(\nu_1+\nu_2)$. Similarly, the exchange integrals were found through the introduction of a well characterized by the oscillator parameter $\nu=(\nu_1+\nu_2)/2$. The numerical values of the various integrals were taken from the tables of Unna.[21] In order to check the foregoing calculation, the Coulomb energy of the di-proton cluster was also evaluated when the oscillator well for the di-proton was the same as that for the alpha-particle clusters. Its Coulomb energy was then nearly equal to the shell-model value for *L*-*S* coupling. Such a result was expected from Elliott's analysis of the various models for light nuclei.[22]

D. $C^{10} \rightarrow B^{10**}$ Transition

The minor branch observed in the beta decay of C^{10} arises from a transition between two members of a $J^\pi=0^+$ isotopic-spin triplet. According to charge independence, the matrix element of this transition is the same as that of the other pure Fermi-allowed transitions. Therefore, its *ft* value should agree with the 3030 ± 12 and 3075 ± 10 sec found for the Al^{26*} and O^{14} decays, respectively.[23]

Although the end point of this transition has not been measured directly, its value can be obtained from the maximum energy of the main positron branch and the separation between the first and second excited states of B^{10}, indicated on Fig. 1. However, this end point, coupled together with the branching ratio measured by Sherr and Gerhart,[2] implies an *ft* value of 2400 ± 400 sec. Since such a *ft* value would be a violation of charge independence,[24] it seems likely that the measurement of the branching ratio is in error.

IV. CONCLUDING REMARKS

Wildermuth and Kanellopoulos[7] have given a simple explanation for the long-tailed charge distribution of Li^6 on the basis of their cluster model of the nucleus. This explanation seems to be confirmed by the calculations of Jackson.[18] However, when their model is applied to the $C^{10}-Be^{10}$ Coulomb energy difference, the analysis presented here shows that it fails. On the other hand, the shell model with an oscillator potential has given a consistent account of the charge distribution in most light nuclei, as deduced both from electron scattering and from the mirror nuclei (including C^{10} and Be^{10}). Li^6 is the notable exception to its success. In this case, however, the basic assumption underlying the shell model, i.e., that the interaction of each nucleon with the other nucleons can be represented by an average potential, is questionable.

ACKNOWLEDGMENTS

The author wishes to thank Professor F. H. Schmidt for his guidance during the course of this work. He is also grateful to Professor E. M. Henley and Professor J. B. Gerhart for their many helpful suggestions.

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[23] J. M. Freeman, J. H. Montague, D. West, and R. E. White, Phys. Letters **3**, 136 (1962).
[24] O. Kofoed-Hansen, Phys. Rev. **92**, 1075 (1953).